Mill Lane Community School

Calculation Policy

Growing, Thriving, Flying

INTRODUCTION

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression. The content is set out in stages under the following headings: addition, subtraction, multiplication and division.

AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding

HOW TO USE THIS POLICY

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Cross reference with the mental maths policy for guidance on key facts, key vocabulary and mental methods
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems

Stages in Addition

Addition - Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to combining two groups of objects, first by counting all and then by counting on from the largest number.

They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.

'You have five apples and I have three apples. How many apples altogether?'

Addition - Stage One

- Given a number, identify one more
- Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign
- Add one- digit and two-digit numbers within 20, including zero
- Solve missing number problems eg 10 + = 16

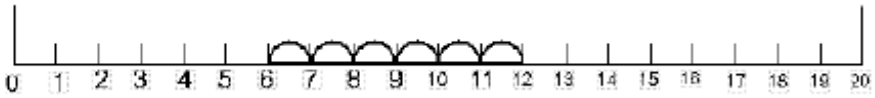
Children will continue to practise counting on from any number e.g. 'Put five in your head and count on four.'

Initially use a number track to count on for addition, counting on from the largest number:

5 + 4 = 9

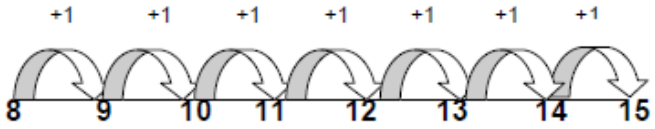
'Put your finger on number five. Count on (count forwards) four.'

Progress to a marked number line: **6 + 6 = 12**



'Put your finger on number six and count on six.'

8 + 7 = 15



Ensure children are confident using a marked number line before moving on to an empty number line. Continue to practice counting on from the largest number for addition with totals within 20.

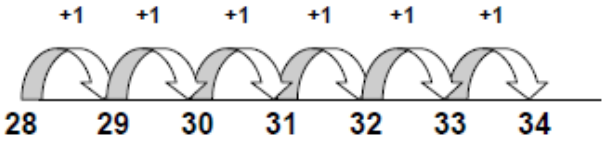
Addition Stage Two

Add numbers using concrete objects, pictorial representations, and mentally including:

- A two digit number and ones
- A two digit number and tens
- 2 two digit numbers
- Three one digit numbers

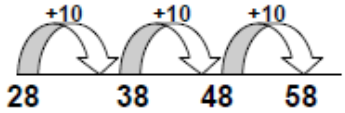
Counting on in ones using an **empty number line**, within 100 ….

28 + 6 = 34



…and in tens

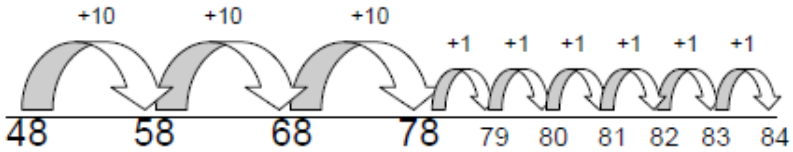
28 + 30 = 58



Use in conjunction with a 100 square to show jumps of tens.

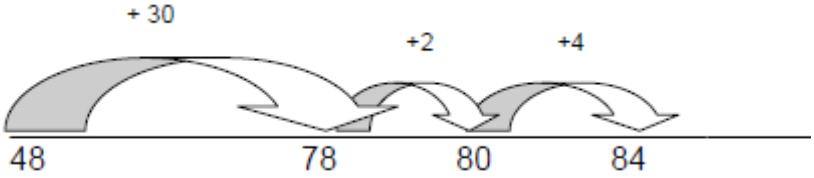
48 + 36 = 84

'Put the biggest number first and then partition the smaller number (36 = 30 + 6) and count on: 48 + 30 + 6'



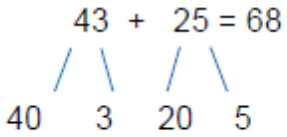
Use in conjunction with 10 square to show jumps of tens and ones.

When children are confident, use more efficient jumps:



Use in conjunction with a 100 square to show jumps of tens and ones/units.

Also use partitioning method to add 2 two digit numbers:



'Partition the numbers into tens and ones / units.
Add the tens together and then add the ones/units together.
Recombine to give the answer.'

40 + 20 = 60
3 + 5 = 8
60 + 8 = 68

Then move on to calculations that bridge the tens:

48 + 36 = 40 + 8 + 30 + 6

This is an alternative way of recording the partitioning method.

40 + 30 = 70
8 + 6 = 14
70 + 14 = 84

Further develop addition with numbers that bridge 100, using a 200 grid to support.

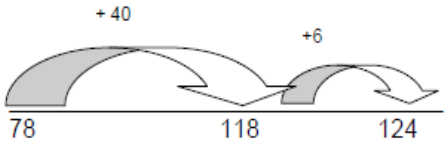
48 + 36 = 84

NB If at any time children are making significant errors, return to the previous stage

Addition Stage Three

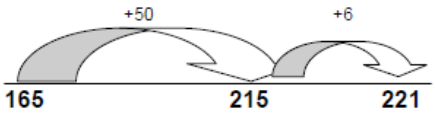
Add numbers with up to three digits, using formal written method of columnar addition. Further develop the use of the empty number line with calculations that bridge 100:

78 + 46 = 124



Use a 200 grid to support counting on in tens and bridging 100 .

165 + 56 = 221



And with addition of a three digit and a two digit number

Further develop the partitioning method with calculations that bridge 100:
85 + 37 = 80 + 5 + 30 + 7

80 + 30 = 110
5 + 7 = 12
110 + 12 = 122

85 + 37 = 122

The partitioning method can also be used with three digit numbers. Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns).

Initially use calculations where it has not been necessary to bridge across the tens and hundreds:

63 + 32 = 95

$$\begin{array}{r} 60 + 3 \\ +\ 30 + 2 \\ \hline 90 + 5 = 95 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then...

$$\begin{array}{rll} & 6\ 3 & \\ + & 3\ 2 & \\ \hline & 5 & (3 + 2) \\ + & 9\ 0 & (60 + 30) \\ \hline & 9\ 5 & \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

This will lead to the formal written method:

$$\begin{array}{rr} & 6\ 3 \\ + & 3\ 2 \\ \hline & 9\ 5 \end{array}$$

Use the language of place value to ensure understanding: 'Three add two equals five. Write five in the units column. 60 add 30 equals 90. Write 9 (90) in the tens column.

Introduce calculations where it is necessary to bridge, returning to an expanded method initially: **68 + 24 = 92**

$$\begin{array}{r} 60 + 8 \\ +\ 20 + 4 \\ \hline 80 + 12 = 92 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then . . . .

$$\begin{array}{rll} & 6\ 8 & \\ + & 2\ 4 & \\ \hline & 1\ 2 & (8 + 4) \\ + & 8\ 0 & (60 + 20) \\ \hline & 9\ 2 & \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

If children are ready, introduce the formal written method, where it is necessary to carry ten from the units to the tens column:

$$\begin{array}{rr} & 6\ 8 \\ + & 2\ 4 \\ \hline & 9\ 2 \\ & {\scriptstyle 1} \end{array}$$

Use the language of place value to ensure understanding: 'Eight add four equals 12. Write two in the units column and 'carry' one (10) across into the tens column. 60 add 20 and the ten that we 'carried' equals 90. Write 9 (90) in the tens column. 92 is the answer.

When children are confident, extend with examples where it is necessary to bridge across the tens and the hundreds:

76 + 47 = 123

$$\begin{array}{r} 70 + 6 \\ +\ 40 + 7 \\ \hline 110 + 13 = 123 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then…

$$\begin{array}{rl} & 76 \\ + & 47 \\ \hline & 13 \quad (7 + 6) \\ + & 110 \quad (70 + 40) \\ \hline & 123 \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

If children are ready introduce the formal written method, where it is necessary to carry across the columns and bridege 100:

76 + 47 = 123

$$\begin{array}{rl} & 47 \\ + & 76 \\ \hline & 123 \\ & {\scriptstyle 1\ 1} \end{array}$$

Use the language of place value to ensure understanding: 'Seven add six equals 13. Write three in the units column and 'carry' one (10) across into the tens column. 40 add 70 and the ten that we 'carried' equals 120. Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100).

If children are confident, further develop with the addition of a three digit number and a two digit number:

178 + 43 = 221

$$\begin{array}{rl} & 178 \\ + & 43 \\ \hline & 221 \\ & {\scriptstyle 1\ 1} \end{array}$$

Addition Stage 4:

Add numbers with up to 4 digits using the formal written method of columnar addition <u>where appropriate.</u> Further develop the formal written method of addition, with three digit numbers. Revisit the expanded method first, if necessary:

176 + 147 = 323

$$\begin{array}{rll} & 176 & \\ + & 147 & \\ \hline & 13 & (7 + 6) \\ + & 110 & (70 + 40) \\ & 200 & (100 + 100) \\ \hline & 323 & \end{array}$$

This will lead in to the formal written method……

176 + 147 = 323

$$\begin{array}{rl} & 147 \\ + & 176 \\ \hline & 323 \\ & {\scriptstyle 1\ 1} \end{array}$$

Use the language of place value to ensure understanding:
'Seven add six equals 13. Write three in the units column and 'carry' one across into the tens column (10). 40 add 70 and the ten that we carried equals 120. Write 2 in the tens column (20) and 'carry' 1 across into the hundreds column (100). 100 add 100 and the 100 that has been carried equals 300. Write 3 in the hundreds column (300).

If children are confident, introduce the addition of a four digit number and a three digit number:

1845 + 526 = 2371

$$\begin{array}{rl} & 1845 \\ + & 526 \\ \hline & 2371 \\ & {\scriptstyle 1\quad 1} \end{array}$$

Continue to develop with addition of 2 four digit numbers and with decimals (in the context of money or measures).

Addition Stage Five:
Add whole numbers with more than 4 digits, including using formal written method (columnar addition). Continue to teach the use of empty number lines with larger number lines (and decimals) as appropriate.
Continue to develop the formal written method for addition with larger numbers (and decimal numbers) and with the addition of three or more numbers:

21848 + 1523 = 23371

Continue to use the language of place value to ensure understanding.

$$\begin{array}{r} 21848 \\ +\ 1523 \\ \hline 23371 \\ {\scriptstyle 1\ \ 1\ \ \ \ \ } \end{array}$$

Use the formal written method for the addition of decimal numbers:
£154.75 + £233.82 = £388.57

$$\begin{array}{r} 154{\cdot}75 \\ +\ 233{\cdot}82 \\ \hline 388{\cdot}57 \\ {\scriptstyle 1\ \ \ \ \ \ } \end{array}$$

Ensure that the decimal points line up.

Continue to practice and apply the formal written method.

Stages in Subtraction

Subtraction – Early Stages:
Children will engage in a variety of counting songs and rhymes and practical activities.
In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.
They will find one less than a given number.

They will begin to relate subtraction to 'taking away' using objects to count how many are left after some have been taken away.

6 – 2 = 4

'Take two apples away. How many are left?'

Children will begin to count back from a given number.

Subtraction Stage One:

- Given a number, identify one less
- Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign
- Subtract one digit and two digit numbers within twenty, including zero
- Solve missing number problems e.g. 20 - ☐ =

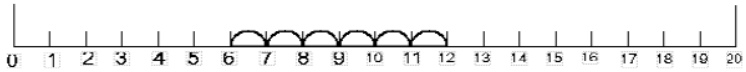
Children will continue to practice counting back from a given number. Initially use a number track to count back for subtraction.

9 – 5 = 4

'Put your finger on number nine. Count back five.'

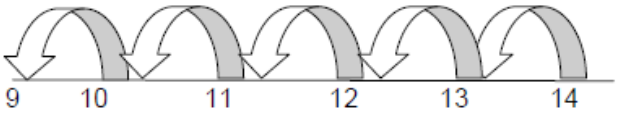
Then progress to a marked number line:

12 - 6 = 6



'Put your finger on number 12 and count back 6.'

14 – 5 = 9

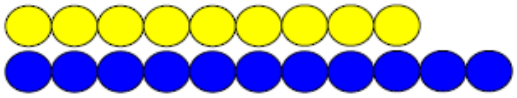


'Put your finger on number 14 and count back 5.'

Continue to practice counting back for subtraction with numbers within 20.

Counting on to find a small difference:
Introduce complementary addition to find differences (only use for small differences).
Count up from the smallest number to the largest to find the difference using resources, e.g. cubes, beads, number track / lines:

11 – 9 = 2



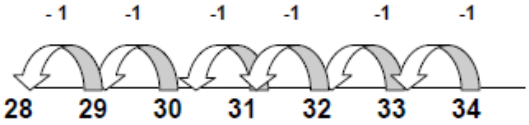
The difference between nine and eleven is two.

Subtraction Stage Two:
Subtract numbers using concrete objects, pictorial representations, and mentally, including:

- A two digit number and ones
- A two digit number and tens
- 2 two digit numbers

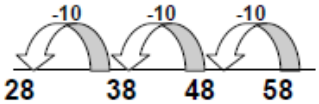
Count back using an empty number line within 100, in ones …..

34 - 6 = 28



And in tens:

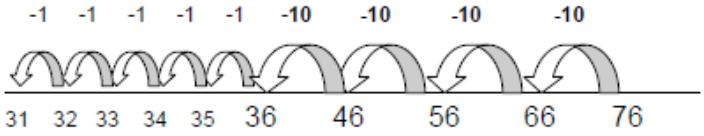
58 - 30 = 28



Use in conjunction with a 100 square to show jumps of tens.

Subtraction, using partitioning, on an empty number line:

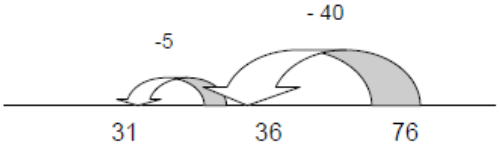
76 – 45 = 31



Use in conjunction with a 100 square to show jumps of tens and ones.

When children are confident, use more efficient jumps:

76 – 45 = 31

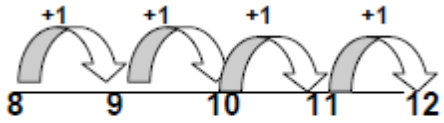


76 – 40 – 5 = 31
Use in conjunction with a 100 square to sh ow jumps of tens and ones.

Counting on

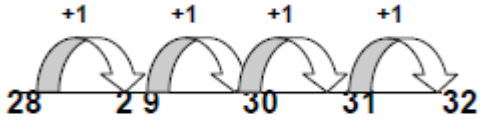
Introduce complementary addition to find differences (only use for small differences). Count up from the smallest number to the largest to find the difference.

12 – 8 = 4



The difference between 8 and 12 is 4.

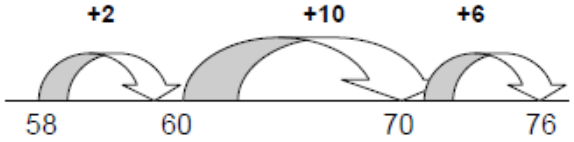
32 – 28 = 4



The difference between 28 and 32 is 4.

If children are confident, further develop this method:

76 – 58 = 18



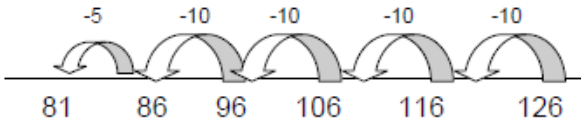
The difference between 58 and 76 is 18.

Further develop subtraction with numbers that bridge 100, using a 200 grid to support.

Subtraction Stage Three:

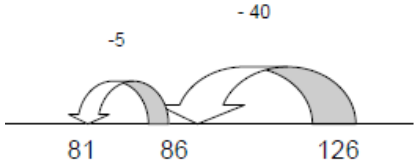
Subtract numbers with up to three digits, using formal written method of columnar subtraction.
Further develop the use of an empty number line with calculations that bridge 100:

126 – 45 = 81



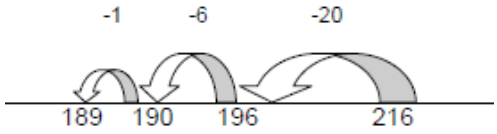
Use a 200 grid to support counting back in tens and bridging 100.

Then use more efficient jumps



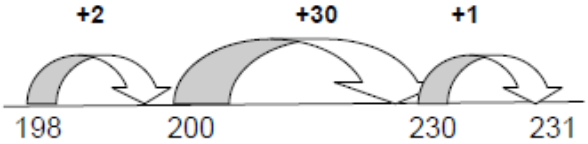
Extend with larger numbers by counting back . . .

216 – 27 = 189



And by counting on to find the difference (small difference):

231 – 198 = 33



Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns). Use two digit numbers when introducing this method, initially:

78 – 23 = 55

70 + 8
−20 + 3
50 + 5 = 55

'Partition numbers into tens and ones/units. Subtract the ones, and then subtract the tens. Recombine to give the answer.'

NB In this example decomposition (exchange) is not required.

You might replace the **+ sign** with the word **'and'** to avoid confusion.

This will lead into the formal written method:

$$\begin{array}{r} 78 \\ -23 \\ \hline 55 \end{array}$$

Use the language of place value to ensure understanding:
'Eight subtract three, seventy subtract twenty.'

Introduce the expanded written method where exchange/decomposition is required:

73 – 27 = 46

$$\begin{array}{r} 70 + 3 \\ -\ 20 + 7 \end{array}$$ becomes $$\begin{array}{r} 60 + 13 \\ -\ 20 + 7 \\ \hline 40 + 6 = 46 \end{array}$$

73 is partitioned into 60+13 in order to calculate 73-27

When children are confident with the expanded method introduce the formal written method, involving decomposition/exchange:

73 – 27 = 46

$$\begin{array}{r} {\scriptstyle 6\ 13} \\ 7\ \not{3} \\ -\ 2\ 7 \\ \hline 4\ 6 \end{array}$$

Use the language of place value to ensure understanding.
'We can't subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'

Use **base ten materials** to support understanding.

If children are confident, extend the use of the formal written method with numbers over 100, returning to the expanded method first, if necessary.

235 – 127 = 108

$$\begin{array}{r} {\scriptstyle 2\ 15} \\ 2\ \not{3}\ \not{5} \\ -\ 1\ 2\ 7 \\ \hline 1\ 0\ 8 \end{array}$$

Use the language of place value to ensure understanding.
In this example it has only been necessary to exchange from the tens column.

Use base ten materials to support understanding.

Subtraction Stage Four:

Subtract numbers with up to four digits using the formal written method of columnar subtraction, where appropriate.
Continue to teach the use of empty number lines with three and four digit numbers, as appropriate.
Continue to develop the formal written method of subtraction by revisiting the expanded method first, if necessary.
Continue to use base ten materials to support understanding.

258 - 73 = 185

$$\begin{array}{r} 200 + 50 + 8 \\ -\ \ \ 70 + 3 \end{array}$$ becomes $$\begin{array}{r} 100 + 150 + 8 \\ -\ \ \ \ \ 70 + 3 \\ \hline 100 + \ \ 80 + 5 \end{array} = 185$$

You might replace the + sign with the word and to avoid confusion.

This leads to the formal written method, involving decomposition….

$$\begin{array}{r} {\scriptstyle 1\ 15} \\ \not{2}\ \not{5}\ 8 \\ -\ \ 7\ 3 \\ \hline 1\ 7\ 5 \end{array}$$

Use the language of place value to ensure understanding.
In this example it has been necessary to exchange from the hundreds column.

Further develop by subtracting a three digit number from a three digit number.

637 – 252 = 385

$$\begin{array}{r} 600 + 30 + 7 \\ -\ 200 + 50 + 2 \end{array}$$

$$\begin{array}{r} 500 + 130 + 7 \\ -\ 200 + \ \ 50 + 2 \\ \hline 300 + \ \ 80 + 5 \end{array} = 385$$

This leads to a formal written method:

$$\begin{array}{r} {}^{5}\not{6}\,{}^{13}\not{3}\,7 \\ -\ 2\,5\,2 \\ \hline 3\,8\,5 \end{array}$$

When children are confident, develop with four digit numbers and decimal numbers (in the context of money and measures).

3625 – 1219 = 2406

$$\begin{array}{r} {}^{1}3\,6\,{}^{15}\not{2}\not{5} \\ -\ 1\,2\,1\,9 \\ \hline 2\,4\,0\,6 \end{array}$$

Subtraction Stage Five:
Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction).
Continue to teach the use of empty number lines with larger numbers and decimals, as appropriate.
Continue to develop the formal written method for subtraction with three and four digit numbers, returning to an expanded method and using base ten materials, if necessary.

503 – 278 = 225

$$\begin{array}{r} 500+\ 0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+\ 8 \\ \hline 200+20+\ 5 \end{array}$$

In this example 503 has to be partitioned into 400+90+13 in order to carry out the subtraction calculation.

This leads in to the formal method:

$$\begin{array}{r} {}^{4}\not{5}\,{}^{9}0\,{}^{13}\not{3} \\ -\ 2\,7\,8 \\ \hline 2\,2\,5 \end{array}$$

There are no tens in the first number (503) so we have to exchange a hundred for 10 tens before we can exchange a ten for ten ones/units

When children are confident extend with larger numbers (and decimal numbers). Return to an expanded method, if necessary.

12731 – 1367 = 11364

$$\begin{array}{r} 1\,2\,{}^{6}\not{7}\,{}^{12}\not{3}\,{}^{11}1 \\ -\ \ 1\,3\,6\,7 \\ \hline 1\,1\,3\,6\,4 \end{array}$$

In this example it has been necessary to exchange from the tens and the hundreds columns.

Introduce subtraction of decimals, initially in the context of money and measures.

£166.25 – £83.72 = £82.53

$$\begin{array}{r} {}^{16}\not{1}\,6\,{}^{5}\not{6}\cdot\,{}^{12}\not{2}\,5 \\ -\ 8\,3\cdot 7\,2 \\ \hline 8\,2\cdot 5\,3 \end{array}$$

Ensure the decimal points line up.

Stages in Multiplication

Multiplication Early Stages:
Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.

'Three apples for you and three apples for me. How many apples altogether?'

Stage One Multiplication:

- Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Count in multiples of twos, fives and tens (to the tenth multiple).

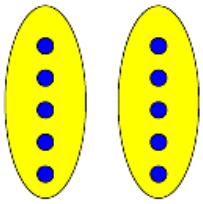
Children will count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts. They will solve practical problems that involve combining groups of 2, 5 or 10 e.g. socks, fingers and cubes.

'Six pairs of socks. How many socks altogether? 2, 4, 6, 8 10, 12.'

'Three pots of 10 crayons. How many crayons altogether? 10, 20, 30.'

Use arrays to support early multiplication.

'Five groups of two faces. How many faces altogether?'
'Two groups of five faces. How many faces altogether?'



'2 groups of 5'

'How many altogether?'

'5 + 5 = 10'

Double five is ten

Continue to solve problems in practical contexts and develop the language of early multiplication, with appropriate resources.

Multiplication Stage Two:

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods and multiplication facts, including problems in contexts
- Show that multiplication of two numbers can be done in any order (commutative)

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x sign to record.

Combining groups (repeated addition)

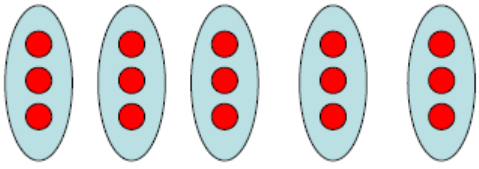
'3 groups of 10 crayons'

'How many crayons altogether?'

'10 + 10 + 10 = 30'

'3 groups of 10' '3 times ten'

'3 x 10 = 30' '10 x 3 = 30'



'5 groups of 3' '5 lots of 3' '3 + 3 + 3 + 3 +3 = 15'

'5 times 3' '3 multiplied by 5' '5 x 3 = 15' '3 x 5 = 15'

Using arrays to support multiplication

6 x 5 = 30

'5 + 5 + 5 + 5 + 5 + 5 = 30 '

'6 rows of 5'

'6 groups of 5'

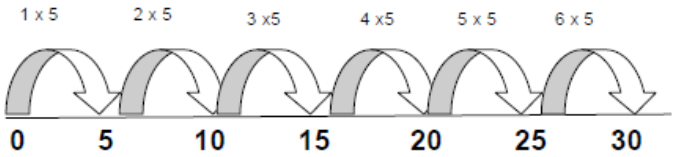
'5 groups of 6'

'5 x 6 = 30'

'6 x 5 = 30'

Using an empty number line

6 x 5 = 30



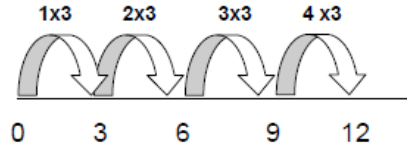
Make the link to repeated addition.

Multiplication Stage Three:

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables (continue to practice 2, 5 and 10).
- Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two digit numbers times one digit numbers, using mental and progressing to a formal written method.

Continue to use numbers lines and arrays to support multiplication, as appropriate.

4 x 3 = 12



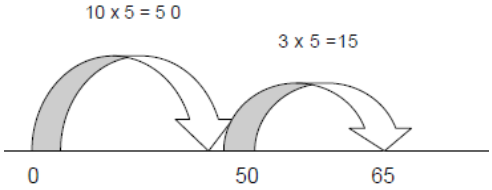
Partitioning method for multiplication of a teen number by a one digit number:

13 x 5 = 65 **(Partition 13 into 10 + 3)**

10 x 5 = 50
3 x 5 = 15

50 + 15 = 65

Demonstrate the partitioning method using a number line: **13. 5 = 65**



Grid method (teen number multiplied by a one digit number): **13 x 8 = 104**

X	10	3
8	80	24

80 + 24 = 104

'Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.'

This will lead into expanded short multiplication: **13 x 8 = 104**

```
10 + 3
x   8
  2 4   (3 x 8)
+ 8 0   (10 x 8)
  1 0 4
```

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication: **13 x 8 = 104**

```
  1 3
x   8
  2 4   (3 x 8)
+ 8 0   (10 x 8)
  1 0 4
```

Formal short multiplication:

```
  1 3
x   8
  1 0 4
    2
```

If children are confident progress to multiplying other two digit numbers by a one digit number.

Stage Four Multiplication:

- Recall multiplication facts for multiplication tables up to 12 x 12
- Multiply two digit and three digit numbers by a one digit number using formal written layout.

Further develop the grid method for two digit numbers multiplied by a one digit number.

36 x 4 = 144

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)

Expanded short multiplication (two digit number by a one digit number):

36 x 4 = 144

```
30 + 6
X   4
  2 4   (4 x 6 = 24)
+ 1 2 0 (4 x 30 = 120)
  1 4 4
```

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication: **36 x 4 = 144**

```
  3 6
x   4
+ 2 4   (4 x 6)
  1 2 0 (4 x 30)
  1 4 4
```

This leads to short multiplication (formal method) of a two digit number multiplied by a one digit number: **36 x 4 = 144**

```
  3 6
x   4
1 4 4
  2
```

Use the language of place value to ensure understanding.
Ensure that the digit 'carried over' is written under the line in the correct column.

If children are confident, continue to develop short multiplication with three digit numbers multiplied by a one digit number. If necessary return to the grid method and /or expanded method first:

127 x 6 = 762

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products)

This leads to expanded short multiplication: **172 x 6 = 762**

```
  1 2 7
x   6
  4 2   (6x7)
+ 1 2 0 (6x20)
  6 0 0 (6x100)
  7 6 2
```

This will lead into short multiplication (formal method):

$$\begin{array}{r} 127 \\ \times\ \ 6 \\ \hline 762 \\ {\scriptstyle 1\ \ 4} \end{array}$$

Multiplication Stage Five:

- Multiply numbers up to four digits by a one or two digit number using formal written method, including long multiplication for two digit numbers.

Introduce multiplication of a two digit number by a two digit number. If necessary return to the grid method and /or expanded method first.

Grid method (two digit number multiplied by a teen number):

23 x 13 = (20 + 3) x (10 + 3) = 299

X	20	3
10	200	30
3	60	9

$$\begin{array}{r} 230 \\ +\ \ 69 \\ \hline 299 \end{array}$$

Add the partial products (200 + 30) + (60 + 9) = 299

Expanded long multiplication: **23 x 13 = 299**

$$\begin{array}{rl} 23 & \\ \times 13 & \\ \hline 9 & (3 \times 3) \\ 60 & (3 \times 20) \\ +30 & (10 \times 3) \\ \hline 200 & (10 \times 20) \\ 299 & \end{array}$$

This leads into….

Compact long multiplication (formal method): 23 x 13 = 299

$$\begin{array}{rl} 23 & \\ \times 13 & \\ \hline +69 & (3 \times 23) \\ 230 & (10 \times 23) \\ \hline 299 & \end{array}$$

Extend to larger two digit numbers:

56 x 27 = (50 + 6) x (20 + 7) = 1512

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Add the partial products (1000+ 120) + (350 + 42) = 1512

Expanded long multiplication (two digit numbers multiplied by two digit numbers): **56 x 27 = 1512**

$$\begin{array}{rl} 56 & \\ \times 27 & \\ \hline 42 & (7x6) \\ 350 & (7x50) \\ +120 & (20x6) \\ \hline 1000 & (20x50) \\ 1512 & \\ {\scriptstyle 1} & \end{array}$$

This expanded method is linked to the grid method.

Compact long multiplication (formal method):

56 x 27 = 1512

```
  5 6
x 2 7
  3 9 4 2   (7x56)
+ 1 1 1 2 0 (20x56)
  1 5 1 2
  1
```

When children are confident with long multiplication extend with three digit numbers multiplied by a two digit number, returning to the grid method first, if necessary.

124 x 26 = 3224

```
    1 2 4
X   2 6
    7 1 4 2 4   (6x124)
+ 2 4 8 0       (20x124)
  3 2 2 4
  1 1
```

Multiplication Stage 6

- Multiply multi digit numbers (including decimals) up to 4 digits by a two digit whole numbers

Continue to practice and develop the formal short multiplication method and formal long multiplication method with larger numbers and decimal. Return to an expanded form of calculation initially.

The grid method (decimal number multiplied by a two digit number):

53.2 x 24 = 1276.8

x	50	3	0·2	
20	1000	60	4	1064·0
4	200	12	0·8	212·8
				1276·8

The formal written method of long multiplication:

```
    5 3·2
x   2 4·0
    2 1 2·8   (53·2 x 4)
1 0 6 4·0     (53·2 x 20)
1 2 7 6·8
```

It is an option to include .0 in this example, but not essential.

The prompts (in brackets) can be omitted if children no longer need them.

Stages in Division

Division Early Stages

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the apples between two people. 'Half of the apples for you and half of the apples for me.'

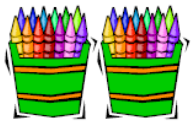
Division Stage One

- Sole one step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Count in multiples of twos, fives and tens (to the 10th multiple).

Children will start with practical sharing using a variety of resources. They will share objects into equal groups in a variety of situations. They will begin to use vocabulary associated with division in practical contexts.

‘Share these eight apples equally between two children. How many apples will each child have?’

‘Share 20 crayons between 2 pots. How many crayons are in each pot?’

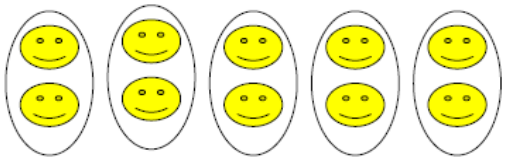


Children will move from sharing to grouping in a practical way.

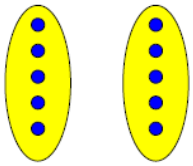
‘Put 20 crayons into groups of 10. How many pots do we need?’

Use arrays to support early di

‘How many faces altogether? How many groups of two?’



‘Five groups of two.’



‘How many groups of 5?’
’10 shared equally between 2 people.’
‘Half of 10 is 5.’

Division Stage Two

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables.
- Calculate mathematical statements for division within the multiplication tables they know and write them using the division (÷) and equals (=) signs.
- Solve problems involving division, using materials, arrays, repeated subtraction, mental methods and multiplication and division facts, including problems in contexts.

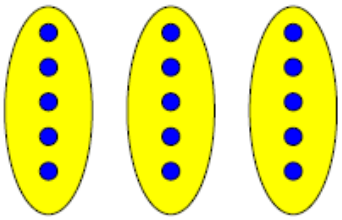
Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the ÷ sign to record, using multiples that they know.

Sharing and grouping:

‘30 crayons shared equally between three pots.’ (Sharing)
‘We have 30 crayons and put ten crayons in each pot.
How many pots do we need?’ (Grouping)

‘30 divided by 10 = 3’
‘30 divided by 3= 10’

30 ÷ 10 = 3
30 ÷ 3 = 10



'How many groups of 5?'
'15 shared equally between 3 people is…?'

'15 divided by 3 equals 5'
'15 divided by 5 equals 3'

15 ÷ 5 = 3
15 ÷ 3 = 5

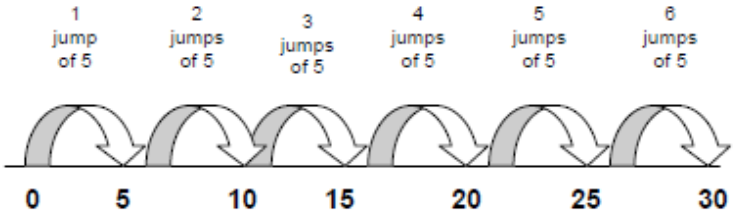
Using arrays to support division:

15 ÷ 5 = 3
15 ÷ 3= 5

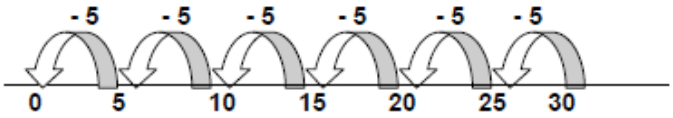
How many groups of 3?
How many groups of 5?
15 shared between 3 people is…?
15 shared between 5 people is…?

15 divided by 5 = 3
15 divided by 3 = 5

When children are ready use an empty number line to count forwards:
30 ÷ 5 = 6 How many jumps of five make thirty?



Also jump back to make the link with repeated subtraction: 30 ÷ 5 = 6 – 'How many groups of 5?'

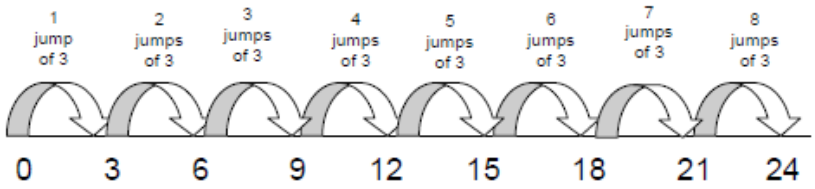


Division Stage Three

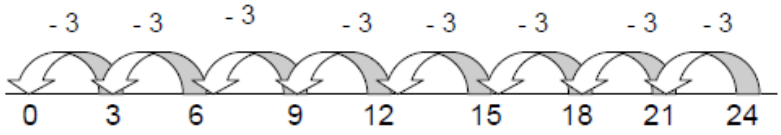
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two digit numbers divided by one digit numbers, using mental and progressing to a formal written method

Continue to use practical resources, pictures, diagrams, number lines, arrays and the ÷ sign to record, using multiples that they know, as appropriate.

24 ÷ 3 = 8 – 'How many threes in 24?'



Also jump back from 24 to make the link with repeated subtraction.



'How many groups of three in 24?'

Introduce the formal layout using multiplication / division facts that the children know: **24 ÷ 3 = 8 can also be recorded as**

$$3\overline{)\,24}\quad 8$$

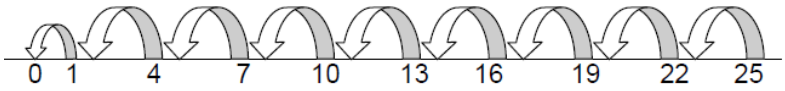
Division Stage Four

- Recall multiplication and division facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to divide mentally
- Divide two digit numbers by a one digit number using formal written layout

Continue using the formal written layout for division using multiplication tables that they know: $8\overline{)\,32}\quad 4$

Continue using the formal written layout, introducing remainders: **25 ÷ 3 = 8 r 1** $3\overline{)\,25}\quad 8\text{ r }1$

this could be modelled using an empty number line, if necessary: 'Eight jumps of 3 and 1 left over.'



Alternatively you could jump forwards in multiples of three from zero to 24 and 'One more makes25'

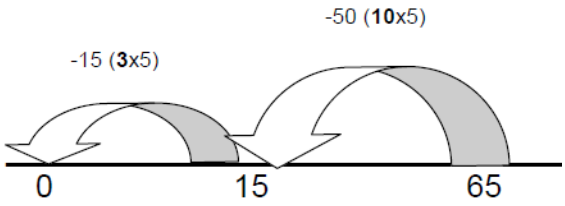
65 ÷ 5 = 13

Division using partitioning (two digits divided by one digit):

65 = 50 + 15 Partition 65 into 50 and 15

50 ÷ 5 = 10
15 ÷ 5 = 3
10 + 3 = 13

Continue to use empty number lines, as appropriate, using multiples of the divisor: **65 ÷ 5 = 13**



98 = 70 + 28 Partition 98 into 70 and 28

98 ÷ 7 = 14

70 ÷ 7 = 10
28 ÷ 7 = 4
10 + 4 = 14

98 ÷ 7 = 14

$$7\overline{)\,70 + 28}\quad 10 + 4 = 14$$

'We have partitioned 98 into 70 and 28 (90 = 70 + 28).

Seven 'goes into' 70 ten times and seven 'goes into' 28 four times.
Ten add four equals 14'

This will lead into the formal written method of short division: **98 ÷ 7 = 14**

98 ÷ 7 = 14

$$7\overline{)\,9^{2}8}\quad 1\,4$$

Use the vocabulary of place value to ensure understanding and make the link to partitioning.

If children are confident develop three digit by one digit using the formal method of short division (no remainders)

Division Stage Five

- Divide numbers up to 4 digits by a one digit number using the formal written method of short division and interpret remainders appropriately for the context.

Continue to practice the formal written method of short division with whole number answers: **184 ÷ 8 = 23**

$$8\overline{)18^{2}4} = 23$$

and with remainders
432 ÷ 5 = 86 r 2

$$5\overline{)43^{3}2} = 86\ r\ 2$$

The remainder can also be expressed as a fraction 2/5
432 ÷ 5 = 86 2/5

Division Stage Six

- Divide numbers up to four digits by a two digit number using the formal written method of short division, where appropriate, interpreting remainders according to the context
- Divide numbers up to four digits by a two digit number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Continue to practice the formal method of short division, with and without remainders, using the language of place value to ensure understanding. **496 ÷ 11 = 45 r 1**

$$11\overline{)49^{5}6} = 45\ r\ 1$$

The remainder can also be expressed as a fraction, 1/11 (the remainder divided by the divisor).
Dividing by a two digit number using a formal method of long division:

$$11\overline{)496} = 45\ r\ 1$$

$$-440 \quad (40 \times 11)$$
$$56$$
$$-55 \quad (5 \times 11)$$
$$1 \quad \text{(remainder)}$$

Multiples of the divisor (11) have been subtracted from the dividend (496)

'40 (lots of 11) + 5 (lots of 11) = 45 (lots of 11)'

'1 is the remainder'

Answer: $45\frac{1}{11}$

Standard short division does not help with the following calculation. However, it can be solved using long division (by repeated subtraction using multiples of the divisor): **144 ÷ 16 = 9**

$$16\overline{)144} = 9$$

$$-64 \quad (4 \times 16)$$
$$80$$
$$-64 \quad (4 \times 16)$$
$$16$$
$$-16 \quad (1 \times 16)$$
$$0$$

Multiples of the divisor (16) have been subtracted from the dividend (144)

'4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16)

There is no remainder'

Children will need to select the most effective method for each calculation / problem they meet, including whether to use the standard formal written method of long division: **432 ÷ 15 = 28 r 12**

$$15\overline{)432} = 28\ r12$$

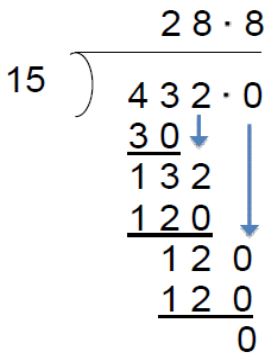
$$300 \quad (20 \times 15)$$
$$132$$
$$120 \quad (8 \times 15)$$
$$12 \quad \text{(remainder)}$$

Multiples of the divisor (15) have been subtracted from the dividend (432)

'20 (lots of 15) + 8 (lots of 15) = 28

12 is the remainder'

This is an alternative way of recording formal long division: **432 ÷ 15 = 28.8**



NB Only teach this method when children are completely secure with the previous method.

The remainder is expressed as a decimal.

Reviewed: January 2024

Signed by Chair of Governors: